Exercise Sheet 7

Due at the beginning of the exercise session at 16:20. A total of at least 50% of all available points is required for the "Studienleistung" (I expect there to be 12 or 13 sheets).

Suppose that I and J are two infinite linear orders. We say that two indiscernible sequences $\bar{a} = \langle a_i : i \in I \rangle$ and $\bar{b} = \langle b_j : j \in J \rangle$ have the same *EM-type* if, for all $n < \omega$, all formulas $\psi(y_0, \ldots, y_{n-1})$, all $i_0 < \ldots < i_{n-1}$ from I, and all $j_0 < \ldots < j_{n-1}$ from J, $\models \psi(a_{i_0}, \ldots, a_{i_{n-1}}) \leftrightarrow \psi(b_{j_0}, \ldots, b_{j_{n-1}})$.

Exercise 1 (Guingona 3.1.4, 2 points)**.** Suppose $\varphi(x; y)$ has alternation rank $\geq n$ as witnessed by an indiscernible sequence $\bar{a}$, and that the sequence $\bar{b}$ has the same EM-type as $\bar{a}$. Show that $\bar{b}$ is also a witness to the fact that $\varphi(x; y)$ has alternation rank $\geq n$.

Exercise 2 (Guingona 3.1.7, 2 points each)**.** Work in an $\mathcal{L}$-language.

(a) Show that if $\varphi(x; y)$ is NIP, then $\neg\varphi(x; y)$ is NIP.

(b) Show that if $\varphi(x; y)$ and $\psi(x; y)$ are both NIP, then $\varphi(x; y) \wedge \psi(x; y)$ is NIP.

(c) Give an example that shows that NIP formulas are not closed under existential quantification.