Exercise Sheet 9

Due at the beginning of the exercise session at 16:20. A total of at least 50% of all available points is required for the "Studienleistung" (I expect there to be 12 or 13 sheets).

Exercise 1 (2 points)**.** Let $\mathbb{N}^*$ be the natural numbers with the reverse ordering, i.e. $0 >_{\mathbb{N}^*} 1 >_{\mathbb{N}^*} 2 >_{\mathbb{N}^*} \ldots$. Show that $\mathbb{N} + \mathbb{N}^*$ is not order-isomorphic to $\mathbb{N}^* + \mathbb{N}$. (Hint: One of these is isomorphic to a familiar linear order.)

Exercise 2 (Jech 5.3, 2 points)**.** Let $(P, <)$ be a linear ordering and let κ be a cardinal. Show that if $|\{x \mid x < y\}| < \kappa$ for all $y \in P$, then $|P| \leq \kappa$.

Exercise 3 (2 points)**.** Let S_α be as defined in the limit case of the proof of Theorem 3.2.1 of the course notes, i.e. $S_\alpha = \bigcup_{n \in \mathbb{N}} (n + f[S_n])$. (Let $S_0 = g_{\beta_0}^{-1}[\beta_0]$ for exactness.) Show that $(S_\alpha, <_{\mathbb{R}})$ is order-isomorphic to $(\alpha, \in)$.