Set Theory
Winter term 2024/2025
Exercises

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Exercise sheet 01 from 18.10.2024

Due at the beginning of the exercise session on 25.10.2024 at 12:00. Any problem is worth 4 points unless specified otherwise. A total of at least 50% of all available points is required for the "Studienleistung" (we expect there to be 12 or 13 sheets).

1. Prove, using only the first 6 axioms from the lecture notes, that there exists exactly one set without any elements (i.e. there exists an x such that $\forall y(y \notin x)$ and any two such sets are equal).

2. Suppose we try defining ordered pairs differently by letting $\langle x, y\rangle_0 := \{x, \{y\}\}$. Show that this definition does not result in an ordered pair, i.e. there are x, x' and y, y' with $x \neq x'$ or $y \neq y'$ such that $\langle x, y\rangle_0 = \langle x', y'\rangle_0$.

3. Prove that there is no set x such that $x \in x$.
Hint: Use the axiom of foundation.

4. Prove without using the powerset axiom that for any two sets A and B the cartesian product $A \times B := \{\langle x, y\rangle \mid x \in A, y \in B\}$ is a set.
Hint: First show using the axiom scheme of replacement that for any $x \in A$, $\{x\} \times B$ is a set and then apply the axiom scheme of replacement again to the function $x \mapsto \{x\} \times B$. How does the resulting set relate to $A \times B$?