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Exercise sheet 04 from 15.11.2024

Due at the beginning of the exercise session on 22.11.2024 at 12:00. Typically, any problem is worth 4 points. A total of at least 50% of all available points is required for the “Studienleistung”.

1. (2 Points) Show that for any regular cardinal $\kappa \geq \omega_1$, the set

$$E^\kappa_\omega := \{\alpha \in \kappa \mid \operatorname{cf}(\alpha) = \omega\}$$

is stationary in κ.

2. Let B be a boolean algebra. An element $a \in B^+$ is called an *atom* if there is no $x \in B$ with $0 < x < a$ (we let $c < d$ if $c \leq d$ and $c \neq d$).

(a) Show that if B is finite, then below every $b \in B$ there is an atom $a \in B$ with $a \leq b$.

(b) Again assume that B is finite. Denote by $A := \{a_1, \ldots, a_n\}$ the set of all atoms in B. Find an isomorphism between B and the powerset algebra on A (see example 26 in the lecture notes).

3. (8 Points) Denote by V_ω the ωth stage of the von Neumann Hierarchy. Which axioms of ZFC does the structure $(V_\omega, \in)$ satisfy?

4. (2 Points) Let $x := \{2, \{3\}\}$. Does the structure $(x, \in)$ satisfy the axiom of extensionality?