Lecturer: Maxwell Levine
Assistant: Hannes Jakob

Exercise sheet 05 from 22.11.2024

Due at the beginning of the exercise session on 29.11.2024 at 12:00. Typically, any problem is worth 4 points. A total of at least 50% of all available points is required for the "Studienleistung".

1. (8 Points) Calculate the following based on the rank of x and y:

(a) The rank of $\{x\}$.
(b) The rank of $\bigcup x$.
(c) The rank of $\mathcal{P}(x)$.
(d) The rank of $x \times y$.

2. Show that whenever $\alpha > \gamma$, the Mostowski-Collapse of $V_\gamma \cup \{\alpha\}$ is equal to $V_\gamma \cup \{\gamma\}$ with the collapsing map given by $\pi(x) = x$ for $x \in V_\gamma$ and $\pi(\alpha) = \gamma$.

Hint: Use $\in$-induction.

3. Let κ be a cardinal with $\mathrm{cf}(\kappa) > \omega$ and $f\colon \kappa \to \kappa$ a *normal* function, i.e. f is *increasing* (i.e. for any $\alpha < \beta < \kappa$, $f(\alpha) < f(\beta)$) and f is *continuous* (i.e. for any limit $\gamma < \kappa$, $f(\gamma) = \sup\{f(\beta) \mid \beta < \gamma\}$).

Show that the set of fixed points of f, the set $C \subseteq \kappa$ consisting of all α with $f(\alpha) = \alpha$, is a club in κ.

Hint: For closure, use continuity. For unboundedness, construct a sequence $(\alpha_n)_{n\in\omega}$ with $f(\alpha_n) = \alpha_{n+1}$ and consider $\alpha := \sup\{\alpha_n \mid n \in \omega\}$.