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Exercise sheet 06 from 29.11.2024

Due at the beginning of the exercise session on 06.12.2024 at 12:00. Typically, any problem is worth 4 points. A total of at least 50% of all available points is required for the "Studienleistung".

1. Suppose κ is an infinite cardinal. Does there exist a $\subseteq$-descending sequence $\kappa \supseteq x_0 \supseteq x_1 \supseteq \ldots$ of length ω such that $|x_n| = \kappa$ for every $n \in \omega$ and $\bigcap_{n\in\omega} x_n = \emptyset$?

2. (6 Points) Suppose $\kappa \geq \lambda$ are infinite cardinals. Consider the following four assumptions:

(a) $\kappa > \lambda$,
(b) $\kappa > \mathrm{cf}(\lambda)$,
(c) $\mathrm{cf}(\kappa) > \lambda$,
(d) $\mathrm{cf}(\kappa) > \mathrm{cf}(\lambda)$.

Which of these assumptions implies that for every $f\colon \kappa \to \lambda$ there exists $\alpha < \lambda$ such that $f^{-1}[\{\alpha\}] := \{\beta \in \kappa \mid f(\beta) = \alpha\}$ has size κ?

In other words, for each individual assumption, considered by itself, either prove the implication or find a counterexample. So (a) is asking whether $\kappa > \lambda$ is enough to imply that for every $f\colon \kappa \to \lambda$ there exists $\alpha < \lambda$ such that $f^{-1}[\{\alpha\}] := \{\beta \in \kappa \mid f(\beta) = \alpha\}$ has size κ, and so on.

3. (6 Points) Determine for the following three formulae whether they are Δ_0, Σ_1, Π_1 or Δ_1 (over ZFC). Try to find an optimal definition. You do not have to prove that it is optimal (we will only later be able to show results of this type).

(a) $y = \mathcal{P}(x)$,
(b) $|y| = |x|$,
(c) x is an ordinal.