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Exercise sheet 07 from 06.12.2024

Due at the beginning of the exercise session on 13.12.2024 at 12:00. Typically, any problem is worth 4 points. A total of at least 50% of all available points is required for the "Studienleistung".

1. Show that the canonical well-ordering of L, restricted to the set $\omega^\omega \cap L$ (i.e. the set of all constructible reals) has ordertype ω_1^L.

Hint: $\omega^\omega \cap L \subseteq L_{\omega_1^L}$.

In what follows, we let M be a countable transitive model of enough ZFC and $(\mathbb{P}, \leq) \in M$ a nonempty partial order.

2. (...)

3. Let G be a filter on $\mathbb{P}$.

(a) Show that G is $\mathbb{P}$-generic over M if and only if $G \cap D \neq \emptyset$ for every open[1] dense subset of $\mathbb{P}$ lying in M.

(b) We call a set $E \subseteq \mathbb{P}$ *predense* if for every $p \in \mathbb{P}$ there is $e \in \mathbb{P}$ such that e and p are compatible. Show that G is $\mathbb{P}$-generic over M if and only if $G \cap E \neq \emptyset$ for every predense subset of $\mathbb{P}$ lying in M.

4. Assume $p, q, r \in \mathbb{P}$ are different. Define the $\mathbb{P}$-name τ by

$$\tau := \{\langle \emptyset, p \rangle, \langle \{\langle \emptyset, q \rangle\}, r \rangle\}\}$$

Let G be a filter which is $\mathbb{P}$-generic over M. Compute all 8 different values of $\mathrm{val}(\tau, G)$ depending on which of p, q, r are in G.

[1] We say that $E \subseteq \mathbb{P}$ is *open* if for all $p \in E$, if $q \leq p$ then $q \in E$.