Set Theory
Winter term 2024/2025
Exercises

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Exercise sheet 11 from 17.01.2025

Due at the beginning of the exercise session on 24.01.2025 at 12:05. Typically, any problem is worth 4 points. A total of at least 50% of all available points is required for the "Studienleistung".

1. Assume $\mathbb{P}$ is atomless. Show that for any filter $G \subseteq \mathbb{P}$ (even outside M), $G \times G$ is not $\mathbb{P} \times \mathbb{P}$-generic over M.

Hint: Either use the Product Lemma or directly define a dense subset of $\mathbb{P} \times \mathbb{P}$ which is not intersected by $G \times G$.

2. Suppose $\mathbb{P}$ is ccc. and $\mathbb{Q}$ is $\aleph_1$-Knaster. Show that $\mathbb{P} \times \mathbb{Q}$ is ccc..

3. Let M be a ctm. Can you find a filter $G \subseteq \mathrm{Add}(\omega)$ which is generic over M and an automorphism π of $\mathrm{Add}(\omega)$ such that $\pi[G]$ is not generic over M?

Hint: By Exercise 1 on sheet 10 π is necessarily outside of M. Define π (using G) such that $\pi[G] \in M$.

4. Let M be a ctm. Suppose $\mathbb{P}$ is ccc. and G is $\mathbb{P}$-generic over M. Let $S \in M$ be such that $M \models S$ is a stationary subset of ω_1. Show that $M[G] \models S$ is a stationary subset of ω_1.

Sketch: By the Forcing Theorem, otherwise there is a $\mathbb{P}$-name τ and a condition $p \in G$ which forces τ to be a club subset of $\check{\omega}_1$ and disjoint from $\check{S}$. Show that $C := \{\alpha \in \omega_1 \mid p \Vdash \check{\alpha} \in \tau\}$ is club in ω_1 (it clearly lies in M). For unboundedness, use the ccc. of $\mathbb{P}$ to define (starting from an arbitrary α_0) an increasing sequence $(\alpha_n)_{n\in\omega}$ such that for each n, $p \Vdash \exists \beta \in \tau(\alpha_n < \beta < \alpha_{n+1})$. Show that $\sup_n \alpha_n \in C$. From the preceding facts, derive a contradiction.