Borel on the questions versus Borel on the answers

Heike Mildenberger *

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Abstract

We consider morphisms (also called Galois-Tukey connections) between binary relations that are used in the theory of cardinal characteristics. In [8] we have shown that there are pairs of relations with no Borel morphism connecting them. The reason was a strong impact of the first of the two functions that constitute a morphism, the so-called function on the questions. In this work we investigate whether the second half, the function on the answers' side, has a similarly strong impact. The main question is: Does the non-existence of a Borel morphism imply the non-existence of a morphism that is only Borel on the answers' side? We give sufficient conditions for an affirmative answer. The results are applied to the unsplitting relations where it has been open whether there is a morphism that is Borel on the answers' side.

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1 Introduction

We work in Vojtáš' framework [10] in which cardinal characteristics of the continuum can be regarded as norms of corresponding relations $\mathbf{A} = (A_-, A_+, A)$ where A_-,$A_+ \subseteq 2^\omega$, $A \subseteq A_- \times A_+$, and the norm of $\mathbf{A}$ is

$$||\mathbf{A}|| = \min\{|\mathcal{Z}| : \mathcal{Z} \subseteq A_+ \wedge \forall x \in A_- \exists z \in \mathcal{Z}\, A(x, z)\}.$$

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A Galois-Tukey connection from a relation $\mathbf{B}$ to a relation $\mathbf{A}$, which we call as in [4] a *morphism from* $\mathbf{A}$ *to* $\mathbf{B}$ (— notice the different direction —), is a pair of functions (α, β) such that

$$\begin{aligned}
&\alpha \colon B_- \to A_-,\\
&\beta \colon A_+ \to B_+,\\
&\forall b \in B_-\ \forall a \in A_+\ (A(\alpha(b), a) \to B(b, \beta(a))).
\end{aligned}$$

This back and forth can, according to Blass [3] be interpreted as follows: The sets A_- and B_- contain questions that are to be answered by suitable elements from the sets of possible answers A_+ and B_+ respectively. The relation $A(x, y)$ says that y is a correct answer to the question x. The task is, given $b \in B_-$, to find a $y \in B_+$ such that $B(b, y)$. Now these questions b are mapped by α to the A-questions "Find an $a \in A_+$ such that $A(\alpha(b), a)$." Any such answer a is mapped back by β to a solution $y = \beta(a)$ of the original task $B(b, y)$. So we consider α as the function on the questions' side and β as the function on the answers' side.

If there is a morphism from $\mathbf{A}$ to $\mathbf{B}$, then $||\mathbf{B}|| \leq ||\mathbf{A}||$, and indeed the proofs of the inequalities usually exhibit morphisms between the corresponding relations.

Like Blass [4], we call inequalities correct if they are true in every model of ZFC, and the other ones incorrect. A result of Yipariki in [11] shows, that there may be morphisms corresponding to incorrect inequalities. These morphisms are not absolute between different models of ZFC, of course, and on the other hand for Borel α, β, $\mathbf{A}$, $\mathbf{B}$ the totality of the two constituents and the back-and-forth condition are both $\mathbf{\Sigma}^1_2$ (boldface, in the parameters coding α and β) hence absolute by Shoenfield's theorem (see e.g. [6, Theorem 98, page 530]).

Most of the morphisms used in the proofs of well-known inequalities between cardinal characteristics e.g. all morphisms used in the proofs of the inequalities in Cichoń's diagram, can be chosen to be Borel functions on Borel domains, which we will call *Borel morphisms*. We consider only relations $\mathbf{A}$, $\mathbf{B}$ whose domains, ranges, and relations itself are Borel subsets of 2^ω respectively $2^\omega \times 2^\omega$.

In [8], the first example of a correct inequality between the norms of two relations without a Borel morphism proving it was given. The older examples for non-existence of Borel morphisms are based upon incorrect inequalities and forcing, see [4] and Section 2 of this paper. Indeed, in [8] the stronger fact that there is no morphism (α, β) with Baire measurable α and arbitrary β was proved. Motivated by this indication of some asymmetry, we are now interested in the complexity of each half of a morphism separately and name the two halves:

We call a morphism (α, β) *Borel morphism on the questions* if α is Borel, and *Borel morphism on the answers* if β is Borel and *semi Borel morphism* if α or β is Borel. We call a morphism *semi Borel morphism* if it is Borel on the questions or Borel on the answers.

In this terminology we can now formulate the questions:

Does the existence of a semi Borel morphism imply the existence of a Borel morphisms? Is there a Borel morphism on the answers for the pairs of unsplitting relations?

With methods of descriptive set theory, we shall give some consistency results for the answers to our questions.

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2 The case of incorrect inequalities

The first kind of results on the non-existence of semi Borel morphisms belongs to a family of pairs of Borel relations $(\mathbf{A}, \mathbf{B})$ such that $||\mathbf{A}|| < ||\mathbf{B}||$ is consistent. The inequality $||\mathbf{A}|| \geq ||\mathbf{B}||$ that would follow from the existence of a morphism of arbitrary complexity is called "incorrect" in [4]. In many cases, namely in the limit steps in the iteration do not destroy the first part of the property $(*)$ below, a model for $||\mathbf{A}|| < ||\mathbf{B}||$ can be gotten by iterating a forcing notion P in V with $(*)$ $\aleph_2$ times with finite support over a model of CH, where:

> $(*)$ For some P-generic g over V: $\forall x \in V[g] \cap A_- \; \exists v \in V \; A(x,v) \wedge g \in B_- \wedge \forall v \in V \; \neg B(g,v)$.

For information on limit steps in forcing interations, see Goldstern [5].

The following theorem is a generalization of the examples given in [4].

Theorem 1 *The existence of a forcing with $(*)$ implies that there are no Borel morphisms on the questions from* $\mathbf{A}$ *to* $\mathbf{B}$ *in V and no Borel morphisms on the answers from* $\mathbf{A}$ *to* $\mathbf{B}$ *in $V[g]$ for g as in $(*)$.*

Proof: : Note that, in constrast to the case of Borel morphisms, we do not know whether the existence of a semi Borel morphism is a $\mathbf{\Sigma}^1_2$-property and hence we have to handle absoluteness in a careful manner.

Suppose that α is Borel and that g is as in $(*)$, and that

$$V \models \forall v \, \exists w \, \forall u \, (A(\alpha(u), v) \to B(u,w)).$$

Fix a $v \in V$ such that $A(\alpha(g), v)$. Then

$$V \models \exists w \, \forall u \, (A(\alpha(u), v) \to B(u,w)).$$

Again, we fix such a $w \in V$. Then, by Shoenfield's absoluteness theorem,

$$V, V[g] \models \forall u \, (A(\alpha(u), v) \to B(u,w)).$$

If we read this in $V[g]$ and insert g for u we get $B(g,w)$, which is a contradiction.

Now suppose β is Borel. For this part see also [10][5.1.5]. Suppose

$$V[g] \models \forall u \, \exists z \, \forall v \, (A(z,v) \rightarrow B(u, \beta(v))).$$

We take $u = g$ and get in $V[g]$ some z such that:

$$V[g] \models \forall v \, (A(z,v) \rightarrow B(g, \beta(v))).$$

Now we take some $v \in V$ such that $A(z,v)$ and get $\beta(v) \in V$ and $B(g, \beta(v))$, a contradiction. $\square$

3 From semi Borel to Borel

In this section, we prove our main results. They apply to situations of correct inequalities. We work mainly in the general setting given in the introduction, and some stronger results are obtained for the sharp unsplitting relations $\mathbf{R}^{\sharp}_n$. Let n^ω be the set of all functions from ω to n and $[\omega]^\omega$ be the set of all infinite subsets of ω. For $n \geq 1$, we have

$$\mathbf{R}^{\sharp}_n = (n^\omega, [\omega]^\omega, \{(f, Y) \, : \, f \text{ is constant on } Y\}).$$

In the following theorem, we use the additivity and the covering number of the ideal $\mathcal{M}$ of sets of first Baire category, also called meager sets:

$$\begin{aligned} \mathrm{add}(\mathcal{M}) &= \min\{\mathrm{card}(\mathcal{Z}) \, : \, \bigcup \mathcal{Z} = \text{ is not meager } \wedge \forall Z \in \mathcal{Z} \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad Z \text{ is meager}\}, \\ \mathrm{cov}(\mathcal{M}) &= \min\{\mathrm{card}(\mathcal{Z}) \, : \, \bigcup \mathcal{Z} = \mathbb{R} \wedge \forall Z \in \mathcal{Z} \; Z \text{ is meager}\}, \end{aligned}$$

and relations $\mathbf{B}$ such that B_- is not meager or such that B_- is the whole space, like in the case of $\mathbf{R}^{\sharp}_3$. There are analogous formulations for the relations $\mathbf{B}$ where B_- is meager but not a measure 0 set: Then the additivity of the ideal $\mathcal{N}$ of Lebesgue measure 0 sets is adequate.

Theorem 2 *If there is a Borel morphism on the answers from* $\mathbf{A}$ *to* $\mathbf{B}$ *and if* $\mathrm{add}(\mathcal{M}) > \aleph_1$ *and if* B_- *is not meager, then there are an open subset* O *of* B_- *and a meager subset of* B_- *such that there is a Borel morphism from* $\mathbf{A}$ *to*

$$(B_- \cap (O \setminus M), B_+, B \cap (B_- \cap (O \setminus M) \times B_+)).$$

Proof: Given a morphism (α', β) with Borel β, any uniformization of the following relation

$$\{(x,y) \, : \, \forall a \in A_+ \; (A(y,a) \rightarrow B(x, \beta(a))\}$$

will yield an α such that (α, β) is a morphism. Since the relation in $\mathbf{\Pi}^1_1$, by Kondô's theorem ([6], page 521), we can choose α to be a $\mathbf{\Pi}^1_1$ function. By a theorem of Sierpiński's ([6], page 520), α is the union of $\aleph_1$ Borel functions α_i, $i \in \aleph_1$. Since $\mathrm{add}(\mathcal{M}) > \aleph_1$ and $B_- = \mathrm{dom}(\alpha)$ is not meager, not all of the $\mathrm{dom}(\alpha_i)$ are meager, say $\mathrm{dom}(\alpha_0)$ is not. As $\mathrm{dom}(\alpha_0)$ is $\mathbf{\Sigma}^1_1$, it has the Baire property and hence contains a basic open set minus a meager set, $O \setminus M$. The pair (α_0, β) is a Borel morphism from $\mathbf{A}$ to $((O \setminus M) \cap B_-, B_+, B \cap ((O \setminus M) \times B_+))$. $\Box$

Remark: Of course, an analogue to Theorem 2 starting with a Borel morphism on the questions can be formulated in an obvious way: The domain of the new β will be A_+ intersected with some open minus a meager set. In general, the questions are not privileged above the answers, however, in the investigated examples, the combinatorics on the questions' side can be handled while the combinatorics on the answers' side does not allow a similar Ramsey type method.

Since there is no Borel morphism on the questions (e.g.) from $\mathbf{R}^\sharp_2$ to $\mathbf{R}^\sharp_3$ restricted to an open $\setminus$ a meager set, cf. [8], and since the domain of $\mathbf{R}^\sharp_3$ is the full 3^ω we get:

Corollary 3 *If* $\mathrm{cov}(\mathcal{M}) > \aleph_1$, *there is no Borel morphism on the answers from* $\mathbf{R}^\sharp_2$ *to* $\mathbf{R}^\sharp_3$.

There are other conditions for the same conclusion:

Theorem 4 *If* $\forall a \in \mathbb{R}\ \omega_1^{L[a]} < \omega_1$ *or, more general, if every* $\mathbf{\Delta}^1_2$ *set has the Baire property and if there is a Borel morphism on the answers from* $\mathbf{A}$ *to* $\mathbf{B}$ *and if* B_- *is not meager, then there is a meager set* M *such that there is a Borel morphism from* $\mathbf{A}$ *to* $(B_- \setminus M, B_+, B \cap (B_- \cap (B_- \setminus M) \times B_+))$.

Proof: By a theorem of Solovay (see [6], page 547), the first premise is a subcase of the second one, so we assume that every $\mathbf{\Delta}^1_2$ set has the Baire property. Suppose that (α, β) is a morphism from $\mathbf{A}$ to $\mathbf{B}$ with second half Borel. α can be chosen $\mathbf{\Pi}^1_1$ by Kondô's theorem. The sets $\alpha^{-1}{}''(\{f \in 2^\omega : f \supseteq s\})$, $s \in 2^{<\omega}$, are $\mathbf{\Delta}^1_2$ and have the Baire property, hence α is Baire measurable. Then we can repeat the well-known argument for Borel functions [9][Exercise 2H.10, page 120] that there is a comeager set on which α is continuous. $\Box$

For the non-trivial morphisms between the sharp unsplitting relations, we get:

Corollary 5 *If* $\forall a \in \mathbb{R}\ \omega_1^{L[a]} < \omega_1$ *or, more general, if every* $\mathbf{\Delta}^1_2$ *set has the Baire property, then there is no Borel morphism on the answers from* $\mathbf{R}^\sharp_2$ *to* $\mathbf{R}^\sharp_3$. $\Box$

Judah and Shelah [7], see also [1, Theorem 9.2.1], showed that every $\mathbf{\Delta}^1_2$ set has the Baire property iff for all $a \in \mathbb{R}$ there is a Cohen generic real over $L[a]$. Since in $L[a]$ there are ω_1 codes for meager sets we get a model where every $\mathbf{\Delta}^1_2$ set has the Baire property just by adding ω_2 Cohen reals to any given model. Also in any model of $\mathrm{cov}(\mathcal{M}) > \omega_1$ every $\mathbf{\Delta}^1_2$ set has the Baire property.

Now we will show that, if there is an inaccessible cardinal, then there is a model of $ZFC + \mathrm{cov}(\mathcal{M}) = \mathrm{add}(\mathcal{M}) = \aleph_1 + \forall a \in \mathbb{R}\ \omega_1^{L[a]} < \omega_1 + \neg CH$. This will show that the hypotheses on cardinal characteristics in Theorem 2 and their corollaries are not necessary.

Theorem 6 *Suppose that $ZFC + \exists\kappa$ inaccessible is consistent. Then the following is consistent: $ZFC + \mathrm{cov}(\mathcal{M}) = \aleph_1 + \omega_1^{L[a]} < \omega_1$ for all $a \in \mathbb{R} + \neg CH$.*

Proof: Suppose that κ is an inaccessible cardinal in V. We take $P_1 =$ the Levy collapse of κ to ω_1, G_1 a P_1-generic filter over V, and P_2 the forcing $B_{\aleph_2}$ for adding simultaneously $\aleph_2$ random reals over $V[G_1]$.

Then $V[G_1][G_2] \models 2^\omega = \aleph_2$ because of the random reals, $V[G_1][G_2] \models \mathrm{cov}(\mathcal{M}) = \omega_1$ because CH holds in V and in $V[G_1]$ and adding random reals with $B_{\aleph_2}$ does not increase $\mathrm{cov}(\mathcal{M})$ (for a proof, see p. 129 of [1]).

We show that $V[G_1][G_2] \models \forall a \in \mathbb{R}\ \omega_1^{L[a]} < \omega_1$: Any real a has a $B_{\aleph_2}$-name $\dot{a}$ over $V[G_1]$. We claim that there is $\alpha < \kappa$ such that $\dot{a} = (||a(n) = 0||^{B_{\aleph_2}} : n \in \omega) \in V[G_1 \restriction \alpha]$.

The boolean values can be chosen as G_δ sets in the measure algebra on $(2^\omega)^{\aleph_2}$ in $V[G_1]$. Hence $\dot{a}$ can be coded by an element of ${}^\omega(\aleph_2 \times \{G_\delta$ sets in $(2^\omega)^{\omega_2}\})$, and hence there are countable sets S_1, S_2 such that $\dot{a}$ is coded in ${}^\omega(S_2 \times \{G_\delta$ sets in $(2^\omega)^{S_1}\})$. So $\dot{a} \in V[G_1 \restriction \alpha]$ for some $\alpha \in \kappa$, because each of the ω components of the code is defined by $G_1 \restriction \alpha$ for some α. κ is inaccessible in $V[G_1 \restriction \alpha]$ and in $V[G_1 \restriction \alpha][a]$ as the algebra belonging to a is c.c.c. in $V[G_1 \restriction \alpha]$.

Since the statement $\forall \gamma \in \kappa\ \gamma^{+\,L[a]} < \kappa$ changes to $\forall \gamma \in \omega_1\ \gamma^{+\,L[a]} < \omega_1$ when κ is collapsed to ω_1, we get:

$$\begin{aligned} V[G_1 \restriction \alpha][a] &\models \forall \gamma \in \kappa\ \gamma^{+\,L[a]} < \kappa, \\ V[G_1 \restriction \alpha][a][G_1 \restriction [\alpha, \kappa)] &\models \forall \gamma \in \omega_1\ \gamma^{+\,L[a]} < \omega_1, \end{aligned}$$

and since the algebra belonging to a is not affected by $G_1 \restriction [\alpha, \kappa)$, it is product forcing and we can switch the last two parts

$$\begin{aligned} V[G_1 \restriction \alpha][G_1 \restriction [\alpha, \kappa)][a] &\models \forall \gamma \in \omega_1\ \gamma^{+\,L[a]} < \omega_1, \\ V[G_1][a] &\models \forall \gamma \in \omega_1\ \gamma^{+\,L[a]} < \omega_1, \\ V[G_1][G_2] &\models \forall a \in \mathbb{R}\ \forall \gamma \in \omega_1\ \gamma^{+\,L[a]} < \omega_1. \end{aligned}$$

$\square$

Remark: The above proof shows that the situation $\forall a \in \mathbb{R}\ \omega_1^{L[a]} < \omega_1$ when obtained by a Levy collapse is preserved when one real is added. If we just have the premise $\forall a \in \mathbb{R}\ \omega_1^{L[a]} < \omega_1$, then there still are some preservation theorems for Souslin c.c.c. forcings and $\mathbf{\Sigma}^1_3$ facts:

Proposition 7 *Suppose that* $\mathbf{A}$ *and* $\mathbf{B}$ *are Borel relations,* β *is* $\Delta^1_1(c)$ *for some* $c \in \mathbf{R}$*. The following statement is a* $\Sigma^1_3(c)$*-formula* $\phi_\beta(c)$*:*

"If there is an α *such that* (α, β) *is a morphism from* $\mathbf{A}$ *to* $\mathbf{B}$*, then there is an* α *such that* (α, β) *is a Borel morphism from* $\mathbf{A}$ *to* $\mathbf{B}$*."*

Proof:

$$\begin{aligned} &[\forall b\, \exists y\, \forall a\, (A(y,a) \quad \to \quad B(b, \beta(a))] \to \\ \exists \alpha\, &[\forall a\, \forall b\, (A(\alpha(b), a) \quad \to \quad B(b, \beta(a)))\ \wedge\ \alpha \text{ is a Borel code }] \end{aligned}$$

is $\Sigma^1_3(c)$. We do not know whether this is an optimal bound for its complexity. $\square$

We use the following fact:

Fact: (Lemma 9.5.4, page 476 in [1]) Assume that $\omega_1^{L[a]} < \omega_1$ for all $a \in \mathbb{R}$. Let G be $\mathbf{P}$-generic over V for some Souslin c.c.c. forcing $\mathbf{P}$. For any $x \in V[G] \cap \mathbb{R}$ and a Σ^1_3 formula $\phi(x)$ with parameters in V,

$$V[x] \models \phi(x) \Leftrightarrow V[G] \models \phi(x).$$

If we add a Hechler generic real G to any model V' of $\forall a \in \mathbb{R}\ \omega_1^{L[a]} < \omega_1 + \neg CH$ (e.g. you the $V[G_1][G_2]$ of Theorem 6), by [2] we get $V'[G] \models \mathrm{cov}(\mathcal{M}) = 2^\omega > \aleph_1, \mathrm{add}(\mathcal{M}) = \aleph_1$, hence by Souslin absoluteness for Σ^1_3 sentences $V'[G]$ is a model of "there is no Borel morphism on the answers from $\mathbf{R}_2^\sharp$ to $\mathbf{R}_3^\sharp$". Also, by Σ^1_3 absoluteness, $V'[G]$ models "if there an Borel morphism with parameter in V' on the answers from $\mathbf{A}$ to $\mathbf{B}$, then there is an open set O and a meager set M such that there is a Borel morphism from $\mathbf{A}$ to $(B_- \cap (O \setminus M), B_+, B \cap (B_- \cap (O \setminus M) \times B_+))$". So our theorems can be transferred to quite different constellations of cardinal invariants.

One can also start form a model of "$\forall a \in \mathbb{R}$ there is a Cohen real over $L[a]$" and then collapse the continuum to ω_1 without adding reals. Then all cardinal invariants are ω_1 but still the theorems on getting Borel morphisms from semi Borel morphisms hold.

4 A general formulation

Here we discuss briefly whether having a Borel morphism from $\mathbf{A}$ to $(B_- \cap (O \setminus M)$, $B_+, B \cap (B_- \cap (O \setminus M) \times B_+))$ implies the existence of a Borel morphism from $\mathbf{A}$ to $\mathbf{B}$.

Definition 8 *A pair of relations* $(\mathbf{A}, \mathbf{B})$ *is called* fairly homogeneous for Borel morphisms *iff for all open sets O in B_- and for all meager sets M in B_- the following holds: if there is a Borel morphism from* $\mathbf{A}$ *to* $((O \setminus M) \cap B_-, B_+, B \cap ((O \setminus M) \times B_+))$, *then there is a Borel morphism from* $\mathbf{A}$ *to* $\mathbf{B}$.

The following are examples among the "natural" relations that can be found in the literature.

1. Any two sharp unsplitting relations. For $n \leq m$, the identity on both components is a Borel morphism from $\mathbf{R}^\sharp_m$ to $\mathbf{R}^\sharp_n$. For $n > m$, for any open set O in n^ω and any meager set M in n^ω there is no Borel morphism from $\mathbf{R}^\sharp_m$ to $(n^\omega \cap (O \setminus M), [\omega]^\omega, ((O \setminus M) \times [\omega]^\omega) \cap R^\sharp_n)$, cf. [8], so the premise in the defining implication is never fulfilled.

2. $(\mathbf{D}, \mathbf{S})$, where $\mathbf{D} = (\omega^\omega, \omega^\omega, \leq^*)$, $\mathbf{S} = ([\omega]^\omega, [\omega]^\omega, \{(X, Y) : \mathrm{card}(X \cap Y) = \mathrm{card}(X \setminus Y) = \omega\}$. As in second half of the first example, the reason for this being an example is, that there is no Borel morphism from $\mathbf{D}$ to $([\omega]^\omega \cap (O \setminus M), [\omega]^\omega, S)$ for any open O and meager M. This is proved as in [4] and it is easy to see that the restriction of $\mathbf{S}$ does not make any difference.

3. A counterexample. If we take a meager set M of Lebesgue measure 1, then the relation $(O \setminus M, \mathcal{N}, \in)$ has norm 1 and there is a Borel morphism from $(O \setminus M, \mathcal{N}, \in)$ into itself (— the identity —) and there is no morphism from $(O \setminus M, \mathcal{N}, \in)$ into $([0, 1], \mathcal{N}, \in)$ of any complexity, because otherwise the latter would have norm 1 instead of the covering number for Lebesgue null sets.

For pairs $(\mathbf{A}, \mathbf{B})$ that are fairly homogeneous for Borel morphisms Theorems 2 and 4 read

Theorem 9 *If there is a Borel morphism on the answers from* $\mathbf{A}$ *to* $\mathbf{B}$ *and if* $\mathrm{add}(\mathcal{B}) > \aleph_1$ *and if B_- is not meager, then there is a Borel morphism from* $\mathbf{A}$ *to* $\mathbf{B}$.

Theorem 10 *If* $\forall a \in \mathbb{R}\ \omega_1^{L[a]} < \omega_1$ *or, more general, if every* $\mathbf{\Delta}^1_2$ *set has the Baire property and if there is a Borel morphism on the answers from* $\mathbf{A}$ *to* $\mathbf{B}$ *and if B_- is not meager, then there is a Borel morphism from* $\mathbf{A}$ *to* $\mathbf{B}$.

We conclude with some open questions:

1. Is Theorem 4 provable in ZFC?

2. Is "there is a Borel morphism on the answers from $\mathbf{R}^\sharp_2$ to $\mathbf{R}^\sharp_3$" consistent?

3. Are there more natural examples $(\mathbf{A}, \mathbf{B})$ that are not fairly homogeneous for Borel morphisms?

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Heike Mildenberger
Mathematisches Institut der Universität Bonn
Beringstr. 1
53115 Bonn, Germany
heike@math.uni-bonn.de