



Seminar:	Whitehead's Problem
Dozentin:	Prof. Dr. Heike Mildenberger
Zeit/Ort:	Mo 14 - 16, SR 318, Ernst-Zermelo-Str. 1
Tutorium:	Dr. Maxwell Levine
Vorbesprechung:	7.2.2022, 13:00 Uhr, in Raum 313 und in https://bbb.uni-freiburg.de/b/hei-fw6-gm7-ijs
Teilnehmerliste:	Mail an heike.mildenberger@math.uni-freiburg.de vor dem 6.2.2022
Web-Seite:	http://home.mathematik.uni-freiburg.de/mildenberger/veranstaltungen/ss22/seminar.html

Content:

In the 1950s the topologist J. H. Whitehead asked the following question: Consider an abelian group A such that every short exact sequence of the form

$$0 \rightarrow \mathbb{Z} \rightarrow B \xrightarrow{h} A \rightarrow 0$$

splits as long as B is abelian, meaning that there is a homomorphism $g: A \rightarrow B$ such that $h \circ g = \text{id}_A$. This assumption is essentially saying that if $h: B \rightarrow A$ is any epimorphism of abelian groups with kernel \mathbb{Z} then B can only be reconstructed from A as a direct sum. All free abelian groups have this property. The question, then, is whether all abelian groups with this property are free.

Surprisingly, Saharon Shelah proved in 1974 that the so-called Whitehead Problem is independent of the Zermelo-Fraenkel Axioms ZFC. More specifically, Shelah showed that a combinatorial principle called \diamond implies that all abelian groups satisfying Whitehead's property are free. On the other hand, if Martin's Axiom holds and $2^{\aleph_0} > \aleph_1$ there is a counterexample.

In the seminar we study this result and the interesting connection it presents between infinitary combinatorics and homological algebra. Such connections are an active area of research today, and there is the possibility of writing a bachelor's or a master's thesis on these topics.

Literature:

- 1.) Paul Eklof, Whitehead's problem is undecidable, Amer. Math. Monthly 83 (1976), no. 10, pp. 775–788.
 - 2.) Paul Eklof and Alan Mekler, Almost free modules, Set-theoretic methods, North-Holland Publishing Co., 2002.
 - 3.) Saharon Shelah, Infinite abelian groups, Whitehead problem and some constructions, Israel J. Math. 18 (1974), 243–256.
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Notwendige Vorkenntnisse:	Vorlesung: Mathematische Logik
Nützliche Vorkenntnisse:	Algebra, Mengenlehre
Studien-/Prüfungsleistung:	Die Anforderungen an Studien- und Prüfungsleistungen entnehmen Sie bitte dem aktuellen Modulhandbuch Ihres Studiengangs.