EXERCISE SHEET 0

(Attendance exercises)

Exercise 1 (Two types of induction).
Using induction, prove the following statements :

i) For $n \geq 6$, we have $3^n > 2n^3$.

ii) Let φ be a formula with exactly n propositional variables. Then the number of binary logical connectors in φ is exactly $n-1$.

Exercise 2 (Another induction).
A straight line always divides a plane into two seperate areas. Into how many areas can a plane be divided at least by exactly n straight lines and into how many areas can it be divided at most?

Exercise 3 (Formalization and Evaluation).
A murder has taken place. The three prime suspects Fischer, Müller and Schmidt are getting interrogated. They put the following to record:

- Fischer: "Müller is the perpetrator and not Schmidt."
- Müller: "If it was Fischer, then he committed the murder together with Schmidt."
- Schmidt: "It wasn't me. It was one of the other two, or even both together."

Formalize the statements of the prime suspects by constructing propositional formulas. Use the letters F, M, S as names for propositional variables which are supposed to stand for „Fischer is the murderer" and so on.
Is it possible that all three suspects are telling the truth? If so, who is the perpetrator? Is it possible that the perpetrator is lying but the others are telling the truth?

Exercise 4 (A questionable proof).
We are going to show that in each bag of gummy bears, there is only one color of gummy bears. This is done by induction over the size of the packaging:

If there is $n = 1$ gummy bear in the bag there is nothing to show.
For the induction step we now assume that for each bag with n gummy bears the statement holds and that we have a bag with $n+1$ gummy bears in front of us. We extract one gummy bear from the packaging. By induction hypothesis the remaining n gummy bears all have the same color. Now we put back the gummy bear that was just removed and extract another one. Again the remaining n gummy bears in the packaging are of the same color by the induction hypothesis. Thus all $n+1$ gummy bears have the same color.

Are you able to find the flaw in this proof? Or is it true that there is no bag of gummy bears with different colors?