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Exercise Sheet 1
(October 19, 2026)

Aufgabe 1. A set (or class) A is called *transitive* if $x \subseteq A$ for all $x \in A$; that is, if $x \in A$ and $y \in x$ imply $y \in A$.

(a) Is $\emptyset$ transitive?

(b) Let x be transitive and $y \subseteq x$. Is $x \cup \{y\}$ transitive?

(c) Let A be a non-empty set of transitive sets. Are $\bigcap A$ and $\bigcup A$ transitive?

Aufgabe 2. A relation R on a set or class A is called *connected* if, for all $x, y \in A$, either $x \in y$, $y \in x$, or $x = y$ holds. Specify a transitive set on which $\in$ is not connected. Can you find an example using only three pairs of curly braces?

Aufgabe 3. For each $n \in \mathbb{N}$, we define the operation $\bigcup^{(n)}$ recursively as follows:

- $\bigcup^{(0)} x = x$,
- $\bigcup^{(n+1)} x = \bigcup(\bigcup^{(n)} x)$.

(a) Let $n \in \mathbb{N}$ be given. Specify a set x such that $\bigcup^{(n+1)} x = \emptyset$ and $\bigcup^{(n)} x \neq \emptyset$.

(b) Find a non-empty set x such that $\bigcup x = x$.

(c) Let x be an arbitrary set. Show that the set $\bigcup\{\bigcup^{(n)} x \mid n \in \mathbb{N}\}$ is transitive.

Aufgabe 4. Let R be an equivalence relation on a proper class A, and let T be a system of representatives for R; that is, T contains exactly one element from each R-equivalence class. [1] Show that either (i) or (ii) holds:

(i) T is a proper class;

(ii) There exists some $w \in A$ such that $w/R := \{u \in A \mid wRu\}$ is a proper class.

Provide examples for cases in which

(a) (i) and (ii) hold,

(b) (i) holds but (ii) does not,

(c) (ii) holds but (i) does not.

[1] Given R, the existence of such a T follows from a strengthening of the Axiom of Choice (AC) known as Global Choice.

Due on Monday October 26, 2026, 10 Uhr, via upload in Ilias.
The path is: Lehrveranstaltungen aus HISinOne –> Wintersemester 2026 –> Mathematisches Institut –> Set Theory – Independence Proofs: Vorlesung | WiSe 26/27