Seminar Nash embedding theorem

Daten: 15. Mai 8:00-13:00 - SR 119
22. Mai 8:00 - 13:00 - SR 119
29. Mai 8:00 - 13:00 - SR 119
2. Juni 14:00 - 18:00 - SR 318

Plan:

1 Introduction and Overview (15.5. Armin)

[2, Sect.1] and [8]

2 Analytic Immersions (15.5 J.Jeßberger)

[5, 1.1] (just use here Cauchy-Kowalevskaya, explanation follows in the next talk) (Lemma 1.1.2 = existence of Fermi coordinates)

3 Analytic Cauchy problems (15.5 Julian)

Cauchy-Kowalevskaya, see e.g. [3]

4 Smooth Immersions (22.5 V.Müller + Nadine)

[5, 1.2]

5 Fixpoint theorems and Dirichlet problems (22.5 M.Amann)

- Fixpoint theorem as used in [5, p.13]
- Dirichlet problem of the Laplacian on Euclidean ball (explicitely vs. minimization) [is used on p.11 in [5]]

6 Global embedding theorem (29.5 Christian+Armin)

[5, 1.3]

7 Nash-Kuiper and Liouville Theorem (2.6 ?)

(I) First version Nash-Kuiper: [7, Kapitel 3, Theorem 3.2].

(II) General Nash-Kuiper: [7, Kapitel 3, Theorem 3.1]. C^1-embeddings of $(\Omega, g) \subset \mathbb{R}^n$ in $\mathbb{R}^m$, $m > n$. "There are C^1-isometric images of $\mathbb{S}^n$ in an arbitrarily small neighbourhood of $\varepsilon\mathbb{S}^n$."

(III) [6, Theorem 2.3.1, p. 36]: Liouville-Theorem/ There are no (smooth) isometric embeddings of $(\Omega, g) \subset \mathbb{R}^n$ in $\mathbb{R}^n$ for $n \geq 3$ (since f has to be a Möbius transform).

Literatur

[1] Adachi, M. *Embeddings and immersions*, vol. 124 of *Translations of Mathematical Monographs*. American Mathematical Society, Providence, RI, 1993. Translated from the 1984 Japanese original by Kiki Hudson.

[2] Andrews, B. Notes on the isometric embedding problem and the Nash-Moser implicit function theorem. In *Surveys in analysis and operator theory (Canberra, 2001)*, vol. 40 of *Proc. Centre Math. Appl. Austral. Nat. Univ.* Austral. Nat. Univ., Canberra, 2002, pp. 157–208.

[3] Folland, G. B. *Introduction to partial differential equations*, second ed. Princeton University Press, Princeton, NJ, 1995.

[4] Grosse, N. Differentialgeometrie I. Skript: `http://home.mathematik.uni-freiburg.de/ngrosse/teaching/DiffGeo_WS-1617_Skript.pdf`.

[5] Han, Q., and Hong, J.-X. *Isometric embedding of Riemannian manifolds in Euclidean spaces*, vol. 130 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 2006.

[6] Iwaniec, T., and Martin, G. *Geometric function theory and non-linear analysis*. Oxford Mathematical Monographs. The Clarendon Press, Oxford University Press, New York, 2001.

[7] Székelyhidi, Jr., L. From isometric embeddings to turbulence. In *HCDTE lecture notes. Part II. Nonlinear hyperbolic PDEs, dispersive and transport equations*, vol. 7 of *AIMS Ser. Appl. Math.* Am. Inst. Math. Sci. (AIMS), Springfield, MO, 2013, p. 63.

[8] Tao, T. Notes on the nash embedding theorem. `https://terrytao.wordpress.com/2016/05/11/notes-on-the-nash-embedding-theorem/`.