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Topology

Problem Sheet 0

This sheet will not be graded!

Exercise 1.

Show that the collection $\mathcal{B}$ of half-open intervals $[a, \infty)$ with a in $\mathbb{R}$ determines a basis of a topology. Is $(0, \infty)$ open in this topology?

Determine whether this topology is coarser or feiner than the euclidean topology.

Exercise 2.

Consider the euclidean topology on the real line $\mathbb{R}$.

a) Describe all subsets of $\mathbb{R}$, which are clopen (that is, both open and closed).

b) Is the euclidean topology 0-dimensional?

Exercise 3.

Given an infinite set X, show that the collection $\mathcal{B}$ of subsets Y with Y empty or $X \setminus Y$ countable determines a topology, the *co-countable topology* on X.

a) Is the co-countable topology comparable to the euclidean topology on $\mathbb{R}$?

b) Describe the interior of the interval $[0, 1]$ as a subset of $\mathbb{R}$ in the co-countable topology.

c) What is the topological closure of $[0, 1]$?

Exercise 4.

Given a subset A of a topological space $(X, \mathcal{T})$, show the following equalities:

a) $(\overline{A})^c = (\overset{\circ}{A^c})$

b) $\overline{A} = A \cup \lim A$.