Prof. Amador Martin-Pizarro — Albert-Ludwigs-Universität Freiburg
Übungen: Xier Ren — Sommersemester 2024

Topology
Problem Sheet 1
Deadline: 23 April 2024, 15h

Exercise 1 (3 Points).

Given subsets A and B of a topological space, show that $\overset{\circ}{(A \cap B)} = \overset{\circ}{A} \cap \overset{\circ}{B}$.

Is $\overset{\circ}{(A \cup B)} = \overset{\circ}{A} \cup \overset{\circ}{B}$ true in general?

Exercise 2 (6 Points).

Let $\mathcal{S}$ be the collection of subsets of $\mathbb{R}$ of the form $(-\infty, a) \cup (b, \infty)$ mit $a < b$.

a) Is $\mathcal{S}$ a basis of a topology?

b) Let now $\mathcal{T}_\mathcal{S}$ be the topology on $\mathbb{R}$ generated by $\mathcal{S}$. Determine whether this topology is T_1 or Hausdorff.

c) Describe the limit points, as well as the interior and the closure of the subset $A = [0, 1] \cup \{2\}$.

d) Is A nowhere dense?

Exercise 3 (8 Points). Consider the collection $\mathcal{B}$ of all subsets of the integers $\mathbb{Z}$ of the form $a + k \cdot \mathbb{Z}$, with $k \neq 0$.

a) Show that $\mathcal{B}$ determines a topology $\mathcal{T}$ on $\mathbb{Z}$ as a basis of $\mathcal{T}$.

b) Show that every non-empty open subset of $\mathbb{Z}$ is infinite. Does $\mathbb{Z}$ have isolated points in this topology?

c) Show that $(\mathbb{Z}, \mathcal{T})$ is 0-dimensional. Conclude that it is Hausdorff.

d) Deduce that there are infinitely many prime numbers.

Hint: Consider the complement of $\{-1, 1\}$.

Exercise 4 (3 Points).

A topological space $(X, \mathcal{T})$ has the *weak Baire property* if $X \neq \emptyset$ and the intersection $\bigcap_{n \in \mathbb{N}} U_n$ of a countable family of open dense subsets U_n is not empty.

Show that $(X, \mathcal{T})$ has the weak Baire property if and only if X is not meager.

Die Übungsblätter können zu zweit eingereicht werden. Abgabe der Übungsblätter im entsprechenden Fach im Keller des mathematischen Instituts.