Nichtkommutative Algebra und Symmetrie
SS 2019 — Ubungsblatt 1

24.4.2019

Informationen zur Vorlesung finden Sie unter:
`http://home.mathematik.uni-freiburg.de/soergel/ss19nkas.html`

Exercise 1.1: Consider the representation $V = \mathbb{R}_+ \oplus \mathbb{R}_-$ of $\mathbb{Z}/2\mathbb{Z}$.

1. How many subrepresentations has V? Is V cyclic?
2. What about $W = \mathbb{R}_+ \oplus \mathbb{R}_+ \oplus \mathbb{R}_-$?

Exercise 1.2: Let $C_n = \mathbb{Z}/n\mathbb{Z}$ be the cyclic group with n elements.

1. Show that representations of C_n is determined by $A \in End(V)$ such that $A^n = \mathrm{id}$
2. Show that all the irreducible (and indecomposable) representations of C_n over $\mathbb{C}$ are of dimension 1.
3. Find all irreducible representations of C_n over $\mathbb{C}$
4. Let $V = \mathbb{C}^n$ and let ρ be the representation of C_n obtained by cycling the coordinates, that is for any $k \in \mathbb{Z}/n\mathbb{Z}$ we have

$$\rho(k)(x_1, x_2, \ldots, x_n) = (x_{k+1}, x_{k+2}, \ldots, x_{k+n \pmod n}).$$

   Write the decomposition of V into irreducible representations.

Exercise 1.3: Let p a prime and C_p the cyclic group with p elements.

1. Find all irreducible representations of C_p over $\mathbb{F}_p$.
2. Find all indecomposable representations of C_p over $\mathbb{F}_p$.
3. (*) Let now G a p-group, that is $|G| = p^k$. Show that G has only one irreducible representation over $\mathbb{F}_p$.

   (Hint: every p-group contains a cyclic group C_p in its center $Z(G)$).

Exercise 1.4: Let (V, ρ) be a representation of a group G.

1. Show that

$$\rho^*(g) : \lambda \mapsto \left(v \mapsto \lambda(\rho(g^{-1})v)\right).$$

   defines a representation of G on V^*.

2. Show that if V is finite dimensional then ρ irreducible $\implies$ ρ^* irreducible.

3. (*) What happens when V is not finite-dimensional?

Bonus Exercise 1.5: There are n knights sitting at a round table. Each of them has a certain amount of food in its plate. Every minute each knight takes half of the serving from each of his/her neighbors. Dinner is served at 7pm. What is the food distribution in the next morning?
(Hint: use exercise 1!)