Nichtkommutative Algebra und Symmetrie
SS 2019 — Ubungsblatt 10

5. Juli 2019

Informationen zur Vorlesung finden Sie unter:
`http://home.mathematik.uni-freiburg.de/soergel/ss19nkas.html`

Exercise 10.1: Let $L(m)$ be the $(m+1)$-dimensional irreducible representation of $SU(2)$. Show that

$$L(m) \otimes L(n) = \bigoplus_{i=0}^{\min\{m,n\}} L(m+n-2i).$$

Exercise 10.2: Let k be a field of characteristic 0, V be a finite dimensional vector space over k and $x, y \in \mathrm{End}_k(V)$. Assume that $(\mathrm{ad}x)^n y = 0$. Show that the generalized eigenspaces of x

$$V_\lambda(x) = \{v \in V \mid (x-\lambda)^n v = 0 \text{ for some } n > 0\}$$

are preserved by y.

Exercise 10.3: Let $\mathfrak{g}$ be a Lie algebra over a field k. A 1-dim. representations of $\mathfrak{g}$ is a hom. of Lie algebras

$$\mathfrak{g} \to \mathrm{End}(k) = k$$

Show that these are precisely the linear maps that vanish on $[\mathfrak{g}, \mathfrak{g}]$.

Exercise 10.4: Let $\mathfrak{g}$ be a nilpotent Lie algebra over an algebraically closed field of characteristic 0. Show that every finite dimensional representation of $\mathfrak{g}$ is the direct sum of its sub-vector spaces which are simultaneously generalized eigenspaces for all the elements of $\mathfrak{g}$.

Exercise 10.5:

- Let k be a field of characteristic 0. Show that $\mathfrak{sl}(k,n)$ is a simple Lie algebra.
- Show that if k of characteristic 2, then $\mathfrak{sl}(k,2)$ is nilpotent.

Exercise 10.6: Show that the Killing form of a nilpotent Lie algebra is identically 0.

Exercise 10.7: Show that if $\mathfrak{g}$ is semisimple, then $\mathfrak{g} = [\mathfrak{g}, \mathfrak{g}]$.