# Nichtkommutative Algebra und Symmetrie
# SS 2019 — Übungsblatt 2

24.4.2019

Informationen zur Vorlesung finden Sie unter:
`http://home.mathematik.uni-freiburg.de/soergel/ss19nkas.html`

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**Exercise 2.1:** Let $k$ be a field. Match the monoids on the left with the corresponding monoid ring on the right (and show that they are isomorphic!).

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|---|---|
| 1. $\mathbb{Z}$ | a $k[x,y]$ |
| 2. $\mathbb{N} \times \mathbb{N}$ | b $k[x,y]/(x^2 - y^3)$ |
| 3. $\{0\} \cup \{n \in \mathbb{N} \mid n \geq 2\}$ | c $k[x,y]/(xy-1)$. |

**Exercise 2.2:** Let $k$ be a field and $C_n$ be the cyclic group with $n$ elements.

1. Show that $kC_n \cong k[x]/(x^n - 1)$.
2. Assume that $(x^n - 1)$ decomposes into linear factors in $k[x]$. Show that all irreducible representations of $C_n$ have dimension 1.
3. Assume further that $n \neq 0$ in $k$. Then show that $kC_n$ is isomorphic as a ring to $\underbrace{k \times k \times \ldots \times k}_{n}$.
4. Show that in this case all representations of $C_n$ are semisimple.

**Exercise 2.3:** Let $m$ be an integer and consider the $\mathbb{Z}$-module $\mathbb{Z}/m\mathbb{Z}$.

1. Write a Jordan-Hölder composition series of the $\mathbb{Z}$-module $\mathbb{Z}/m\mathbb{Z}$.
2. Show that $\mathbb{Z}/m\mathbb{Z}$ is a semisimple $\mathbb{Z}$-module if and only if $m$ is square-free (that is, if $p^2$ does not divide $m$ for every prime $p$).

**Exercise 2.4:** Let $k$ be an algebraic closed field. Show that a $k[x]$-module $M$ is semisimple if and only if the action of $x$ on $M$ is diagonalizable.

**Exercise 2.5:** For a ring $A$ we denote its center by

$$Z(A) = \{x \in A \mid ax = xa \text{ for any } a \in A\}.$$

Let $k$ be a field. Let $G$ be a finite group with group ring $kG$. Let $C_1, C_2, \ldots, C_n$ be the conjugacy classes of $G$. Let $x_i = \sum_{g \in C_i} g \in kG$.

1. Show that $x_i \in Z(kG)$.
2. Show that $\{x_i\}_{1 \leq i \leq n}$ is a basis of $Z(kG)$ as a $k$-vector space.