Nichtkommutative Algebra und Symmetrie
SS 2019 — Übungsblatt 4

22. Mai 2019

Informationen zur Vorlesung finden Sie unter:
`http://home.mathematik.uni-freiburg.de/soergel/ss19nkas.html`

Exercise 4.1: Let k be a field and G be a group. Let V and W be representations of G.

1. Show that
$$g \cdot (v \otimes w) = gv \otimes gw \text{ for all } g \in G,\ v \in V \text{ and } w \in W$$
defines a representation of G on $V \otimes_k W$.

2. Assume that V and W are finite dimensional. Show that
$$\chi_{V \otimes W}(g) = \chi_V(g)\chi_W(g)$$
for all $g \in G$.

Exercise 4.2: Let Q_8 be the quaternion group with 8 elements $\{1, -1, i, -i, j, -j, k, -k\}$ where the multiplication is defined as follows:
$$(-1)^2 = 1, \quad (-1)x = -x \text{ for all } x$$
$$i^2 = j^2 = k^2 = -1$$
$$ij = k,\ jk = i,\ ki = j$$
$$ji = -k,\ kj = -i,\ ik = -j$$
Compute the character table of Q_8.
(Hint: (-1) is central, and $Q_8/\{\pm 1\} \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$).
Compare it with the character table of D_4. Does the character table determine the group?

Exercise 4.3: Let G be a finite group and let χ be the character of a finite dimensional representation of G over $\mathbb{C}$. Show that
$$N := \{g \in G \mid \chi(g) = \chi(1)\}$$
is a normal subgroup of G.
Hint: $\chi(g)$ is the sum of (hom many?) roots of unity.

Exercise 4.4: Recall the character table of the symmetric groups S_3 and S_4 from Exercise 3.2. Use Satz 3.1.2 from the lecture notes to identify each irreducible representation of S_3 and S_4 with the corresponding Young diagram.