# Nichtkommutative Algebra und Symmetrie
# SS 2019 — Übungsblatt 6

29. Mai 2019

Informationen zur Vorlesung finden Sie unter:
`http://home.mathematik.uni-freiburg.de/soergel/ss19nkas.html`

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**Exercise 6.1:** Consider the following groups:

$$GL(n, \mathbb{R}), \quad SL(n, \mathbb{R}) = \{M \in GL(n, \mathbb{R}) \mid \det(M) = 1\},$$

$$O(n, \mathbb{R}) = \{M \in GL(n, \mathbb{R}) \mid {}^tMM = Id_n\}.$$

Which one of them is compact?
**Bonus:** What about the groups $O(n, \mathbb{C})$ and $U(n, \mathbb{C}) = \{M \in GL(n, \mathbb{C}) \mid {}^t\overline{M}M = Id_n\}$?

**Exercise 6.2:** Write down a Haar measure for the group $\mathbb{C}^*$.

**Exercise 6.3:** Write down a Haar measure for $GL(n, \mathbb{C})$. Is it right invariant?
Hint: Recall that if $\mathcal{L}$ is the Lebesgue measure on $\mathbb{R}^m$ and $A \in GL(m, \mathbb{R})$, then $\mathcal{L} \circ A = \det(A) \cdot \mathcal{L}$. Embed $GL(n, \mathbb{C}) \subset \mathbb{R}^{2n^2}$. What is determinant of the linear map induced by the multiplication of a matrix on $\mathbb{R}^{2n^2}$?

**Exercise 6.4:** Let $G$ be an abelian compact group. Show that all the indecomposable finite dimensional represenations over $\mathbb{C}$ of $G$ are of dimension one.

**Exercise 6.5:** Write down all the finite-dimensional irreducible representations over $\mathbb{C}$ of the group $S^1 := \{z \in \mathbb{C}^* \mid |z| = 1\}$.