# Nichtkommutative Algebra und Symmetrie
# SS 2019 — Übungsblatt 7

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Informationen zur Vorlesung finden Sie unter:
`http://home.mathematik.uni-freiburg.de/soergel/ss19nkas.html`

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**Exercise 7.1:** Let $G$ be a group and $V$ a representation of $G$ over $\mathbb{R}$. Let $\mathbb{K} = \mathbb{C}$ or $\mathbb{H}$.

1. Show that $\mathrm{End}^G_{\mathbb{K}}(\mathbb{K} \otimes_{\mathbb{R}} V) \cong \mathbb{K} \otimes_{\mathbb{R}} \mathrm{End}^G_{\mathbb{R}}(V)$.
2. Show that extension of scalar gives bijection between irreducible representations over $\mathbb{R}$ of real type and representations over $\mathbb{C}$ (or over $\mathbb{H}$) of real type.

**Exercise 7.2:** Let $G$ be a finite group.

1. Show that any representation over $\mathbb{H}$ admits a non-trivial $G$-invariant quaternionic scalar product.
2. Let $\langle -, - \rangle$ a $G$-invariant quaternionic scalar product on $V$. Let $jot : \mathbb{H} \to \mathbb{C}$ the map defined by $jot(a + jb) = b$ for all $a, b \in \mathbb{C}$. Show that $jot(\langle -, - \rangle)$ is a $\mathbb{C}$-linear symplectic $G$-invariant bilinear form on $res^{\mathbb{C}G}_{\mathbb{H}G}(V)$.

**Exercise 7.3:** Let $G$ be a finite group. For a character $\chi$ let $FS(\chi)$ be the Frobenius-Schur indicator:

$$FS(\chi) = \frac{1}{|G|} \sum_{g \in G} \chi(g^2)$$

Show that

$$\sum_{\chi \text{ irred.}} FS(\chi)\chi(g) = |\{h \in G \mid h^2 = g\}|$$

Hint: Consider the class function $\theta(g) = |\{h \in G \mid h^2 = g\}|$.

**Bonus Exercise 7.4:** Let $G$ be a finite group of odd order. Show that the trivial representation is the only irreducible complex representation of $G$ of real type.